Influence of Turbulence on Mass Transfer Between a Liquid and a Solid Sphere

Electrochemical measurements of overall mass transfer rates from a single sphere have been performed in a highly turbulent liquid. The increase in mass transfer rate due to turbulence has been shown to be more important at high Reynolds number and high turbulence intensity. Various equations of correlation have been proposed between the overall Sherwood number, the Reynolds number and turbulence intensity, the simplest one being

$$Sh = 6.82\ Re^{0.559}\ Tu^{0.069}$$

A correct agreement has been found with previous heat or mass transfer results between a gas and a sphere.

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SCOPE

Many chemical processes are concerned with mass or heat transfer between solid particles and a fluid in highly turbulent flow. From a fundamental point of view such situations are extremely complex and a limited number of informations can be found in the literature on the mechanism of turbulent transport or even on the gross influence of turbulence on the transfer rates.

The more often, experimental works have been performed in the case of a single particle, of simple form, sphere or cylinder, for heat transfer in a gas phase. Even for these simple cases, data from different authors are scattered and a few general conclusions can be drawn. It is generally acknowledged that turbulence enhances transport phenomena and that the increase is more important at high Reynolds numbers. The most significant parameter seems to be the turbulence intensity.

Here new data are presented in the case of a single sphere, for mass transfer in a highly turbulent liquid flow. Mass transfer rates have been determined by an electrochemical method and turbulence intensities—up to 30%—by hot wire anemometry.

CONCLUSIONS AND SIGNIFICANCE

The turbulence of the liquid flow has been produced by a porous plate. The decay of the intensity of turbulence has led to values varying from 0.30 to 0.04 when the distance behind the porous plate has been increased from 0.03 to 0.50 m. Turbulence intensity has been proved not to be a function of the liquid time-smoothed velocity and the well-known local isotropy hypothesis has been verified to within 15%.

Mass transfer rates have been improved by turbulence, this enhancement increasing with turbulence intensity and the sphere Reynolds number. Several different forms of equations have been tested and have proved to be valuable in correlating experimental data in the range

$$330 < Re < 1720 \text{ and } 0.04 < Tu < 0.30$$

The simplest form of equation is $Sh = 6.82\ Re^{0.559}\ Tu^{0.069}$

When changing the sphere diameter from 5 to 10 mm, no variations in the mass transfer rate have been observed, which suggests that the ratio of the integral length scale to the sphere diameter, though not measured, has no significant influence.

A comparison with previous results in a liquid or a gas and as well for heat transfer as for mass transfer suggests that the influence of turbulence depends neither on the nature of the transported entity nor on the nature of the fluid phase.

OUTLINES OF PREVIOUS WORKS

The effects of the velocity of a laminar stream on mass transfer from a solid sphere have been widely investigated. The situation is completely different in turbulent flows and the effects of velocity fluctuations have not yet been thoroughly analyzed. Indeed, since the pioneering work of Maisel and Sherwood (1950) relatively few experimental works have been devoted to the study of turbulence influence on mass transfer from a sphere (Hsu and Sage, 1957; Brown et al., 1958, 1961; Venezian et al., 1962; Galloway and Sage, 1967, 1968; Endoh et al., 1972; Brauer, 1973; Galloway and Seid, 1975).

Surveying the set of previous works concerned with the influence of turbulence on heat or mass transfer from cylinders or spheres, a few general results can be deduced.

Practically all indicate an increase in the transfer rate with an increase in the turbulence intensity, except Hsu and Sage (1957) and Yuge (1960) that found a negligible influence. It is then generally recognized that the increase in transfer rate is more important at high Reynolds numbers.

Most authors have not investigated the influence of the integral length scale of turbulence or claim that it presents no effects on the mass transfer rate (Maisel and Sherwood, 1950; Brown et

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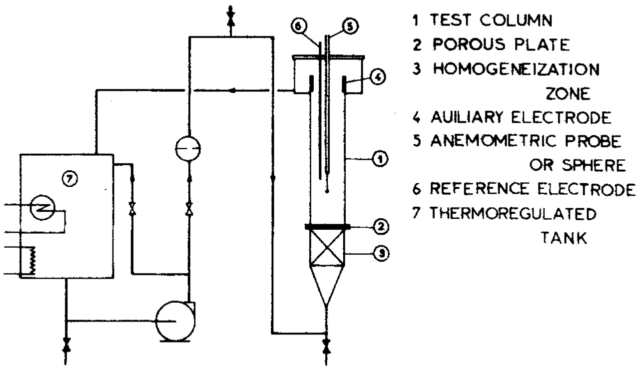

Figure 1. Diagrammatic arrangement of apparatus.

al., 1958; Gostkowski and Costello, 1970; Kestin and Wood, 1971). Following Raithby and Eckert (1968) this would be true only at low Reynolds numbers. Galloway and Sage (1968) put evidence of an increase in transfer rate with an increase in the ratio of the sphere diameter D to the integral length scale of turbulence Λ, when $D/\Lambda < 1$. When $D/\Lambda > 4$ they found no more influence of this parameter.

A large number of experimental correlations for heat or mass transfer from a sphere or a cylinder to a turbulent fluid flow, superimpose turbulent effects on the influence of the time-smoothed stream velocity, usually described by a $Re^{1/2}$ function. These correlations generally involve the dimensionless groups $Re\ Tu$ (Endoth et al., 1972; Lavender and Pei, 1967) or $Re^{1/2}\ Tu$ (Galloway and Sage, 1967; Kestin and Wood, 1971), showing evidence that turbulence effects are closely dependant on the Reynolds number.

General correlations, based on a large number of data proposed by various authors have been developed by Galloway and Sage (1967) then Brauer (1973); these equations are not completely satisfactory because of the large scatter of the data.

It is important to note that the previous preceding studies have been realised in gas phase and with low turbulence intensities. Only the work by Galloway and Seid (1975) has been realised in a liquid flow, in a stirred vessel, but the fluid flow turbulent properties have not been measured directly but approximately deduced from previous results. The conclusions drawn by these authors were that the results with a liquid were quite similar to those obtained with a gas and could be represented by the Galloway and Sage (1967) equation.

EXPERIMENTAL

The experimental equipment (Figure 1) is composed of a 94 mm in diameter vertical column, closed in its bottom by a fixed bed of glass spheres working as a homogeneisation zone. Turbulence is generated by a polyethylene porous plate which is 3.2 mm in thickness, 35% in porosity and 80 μm in average pore diameter.

The liquid is maintained at 20 ± 0.1°C and the flow rate measured by use of an orifice.

With the exception of the electrodes, no metallic material has been used even in the pump. The sphere has been fixed on a thin electrically insulated tube at its rear stagnation point where this support has been shown (Raithby and Eckert, 1968) to present the least hydrodynamical influence.

Overall mass transfer coefficients from the sphere have been measured using a well-known electrochemical method, based on the diffusion controlled reduction of potassium ferricyanide (Mizushina, 1970). The liquid is an equimolecular mixture, 2.10^{-3} N, of potassium ferricyanide and potassium ferrocyanide in 0.5 N sodium hydroxide.

The cathode is a brass sphere coated with a 5μm electrodeposited gold film. The anode is a large stainless steel plate. The origin of electric potentials is given by a saturated calomel electrode. The spherical cathode is polarised by a Tacussel PRT-10-05 potentiostat.

The intensities of turbulence have been measured with a DISA hot-film anemometer constituted by a 55 M01 unit with a standard bridge, a D35 RMS voltmeter, a 55 D31 digital voltmeter, a 55 M25 linearizer and a 55 R11 fiber film probe. This probe is composed of a cylindrical quartz core, covered by a thin metallic film, itself coated with a 2 μm insulating quartz film.

The hot-film anemometer has been calibrated using a special equipment, already described (Bertrand and Couderc, 1978) with a rotating arm which moves the probe in a channel filled with still water.

For the measurements in the column, at each point, the probe has been oriented in three orthogonal directions and for each direction the mean and R.M.S. values have been recorded. The time smoothed vertical velocity and the three components of the RMS value of the velocity fluctuations can then be determined, using the classical hypothesis that the time-smoothed velocity is proportional to the time-smoothed value of the potential difference in the probe; the details of the treatment are given by Sandoval (1978).

Knowing the three RMS values of the velocity fluctuations components, the intensity of turbulence Tu has been determined:

$$Tu = \frac{[1/3(\overline{v_z'^2} + \overline{v_r'^2} + \overline{v_\theta'^2})]^{1/2}}{\bar{v}}$$

RESULTS

Turbulence Behind the Porous Plate

The anemometric measurements have been performed in distilled water because the insulating coating of the probe was damaged by sodium hydroxide.

A flat velocity profile has been observed all along the column, showing that the homogeneizing fixed bed is efficient and that, in the test section, the building of boundary layers along the wall is not important. A good agreement has been obtained between the time-smoothed velocity profiles obtained by hot film anemometry and the values of the flow rate deduced from the orifice measurements.

The time-smoothed velocity has been varied from 0.05 to 0.20 m/s and the variations of the intensity of turbulence as a function of the distance from the porous plate have been recorded.

It must be noticed first, that the differences between the three components of the RMS value of the velocity fluctuations remain very weak, on a mean 14.4% between the θ and z components and 14.0% between the r and z components. This shows that the turbulence produced by the porous plate is quite isotropic which is in good agreement with previous works (Hinze, 1975).

The results of 120 experiments for 12 different time-smoothed velocities are presented on Figure 2.

Turbulence intensity is not a function of the time-smoothed velocity and depends only on the distance from the plate. It can

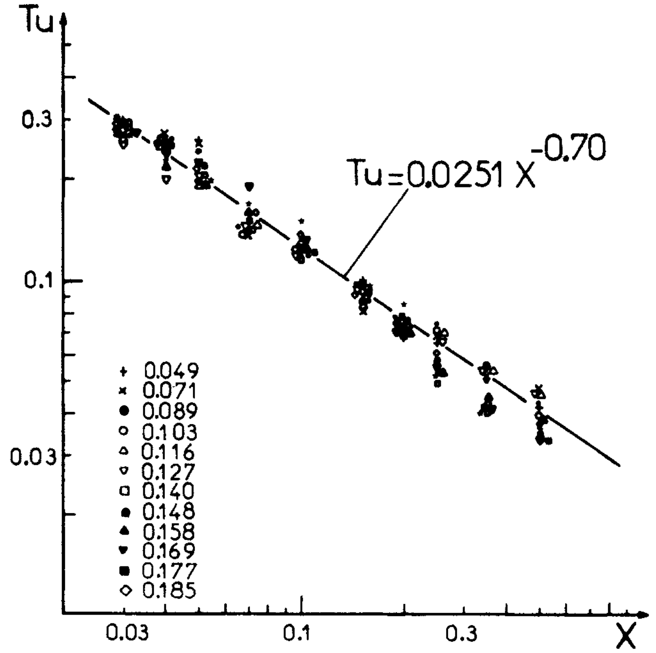

Figure 2. Relationship between turbulence intensity and distance from porous plate.

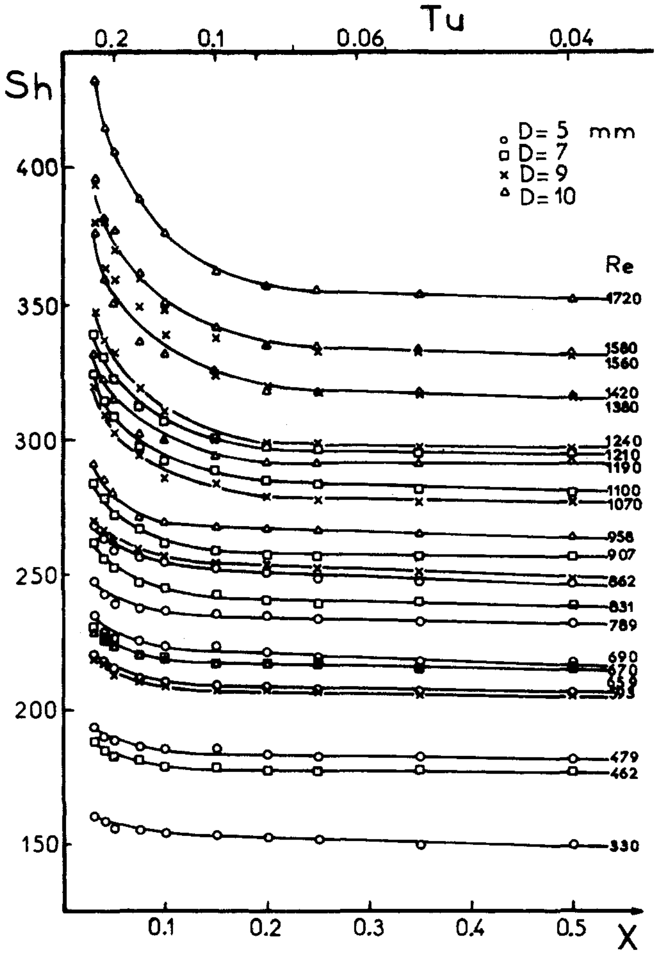

Figure 3. Sherwood number plotted against distance from porous plate (and turbulence intensity) at various Reynolds numbers.

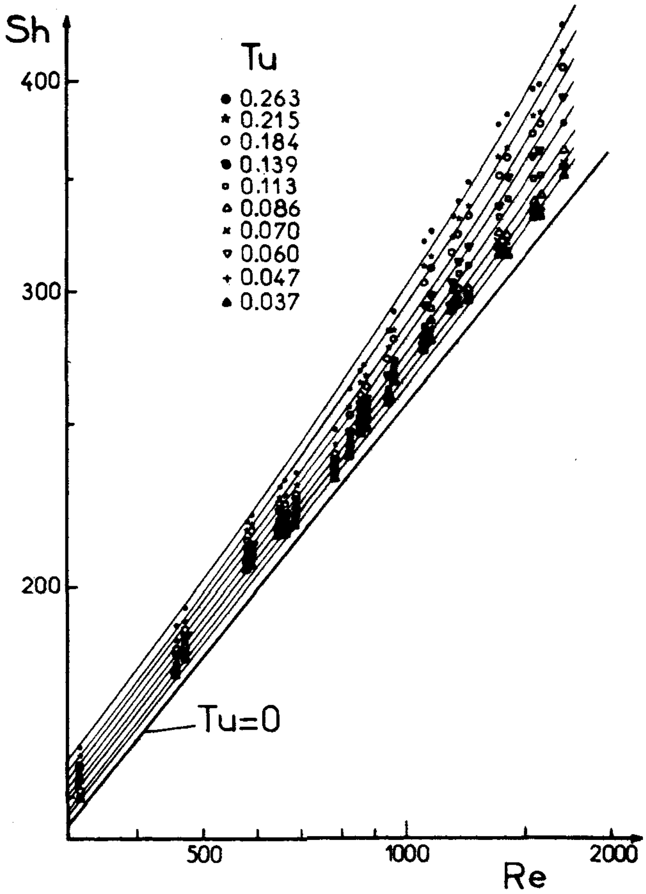

Figure 4. Sherwood number plotted against Reynolds number at various turbulence intensities.

be pointed out that with this apparatus turbulence intensities up to 0.30 can be reached at 0.03 m downwards the porous plate.

The 120 experimental results obtained can be represented by the following correlation

$$Tu = 0.0251X^{-0.70}$$

with a mean deviation of 12.2%.

It is important to note that the exponent of X, -0.70 is very close to the value $-5/7$, theoretically derived by Frenkiel (1948) and Kolmogoroff (1941) and experimentally verified by Baines and Peterson (1951).

Turbulent Mass Transfer

The mass transfer experiments have been performed in the same hydrodynamical and geometrical conditions as velocity measurements, but distilled water has been replaced by an aqueous solution of sodium hydroxide and potassium ferro and ferricyanide. Transposition of the velocity results obtained with distilled water to this solution has been made by multiplying the turbulence intensity by the ratio of kinematic viscosities to the power $-5/2$. In this study, the kinematic viscosities are not very different and this correcting factor, suggested by Hinze (1975), always remained lower than 25%. Four different gold coated spheres, 5, 7, 9 and 10 mm in diameter, have been used and moved in the column from 0.03 to 0.5 m downwards the porous plate in a flow field with constant time-smoothed velocity but decreasing intensity of turbulence. The results are presented on Figures 3 and 4.

Figure 3 shows that the mass transfer rate increases when the distance to the porous plate decreases, or when the intensity of turbulence increases. At low Reynolds numbers, $Re < 500$, the variations are only noticeable for $Tu > 0.15$. On the contrary at higher Reynolds numbers, $Re > 1000$, the effects of turbulence are more important and appear at lower Tu, about 0.07.

Figure 3 also demonstrate that a change from 5 to 10 mm in the sphere diameter has no effect on the increase of transfer rate due to turbulence. Though the integral length scale of turbulence Λ has not been measured, this result suggests that the ratio D/Λ has no noticeable influence on the mass transfer rate.

Experimental results can be represented, as a function of Reynolds number, by various curves corresponding to different values of Tu (Figure 4); the slope of the curves increases with the Reynolds number and turbulence intensity. It can be noted that at low turbulence intensity, the data obtained here are in

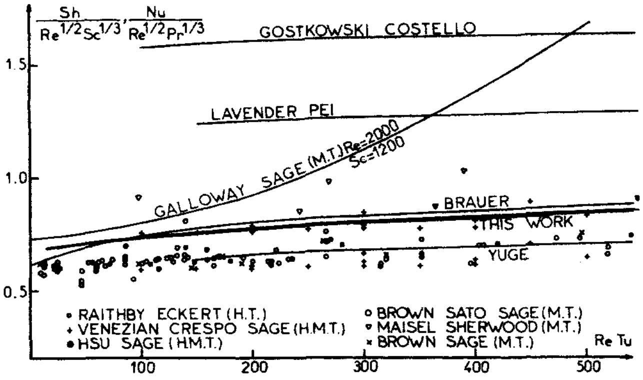

Figure 5. Comparison with other available heat and mass transfer data.

TABLE 1. CORRELATIONS OF POWER LAW.

	Standard Deviation %
(1) $Sh = 6.82\ Re^{0.559}\ Tu^{0.069}$	2.56
(2) $Sh/Re^{\frac{1}{2}} = 8.06\ (Re^{\frac{1}{2}}Tu)^{0.073}$	2.69
(3) $Sh/Re^{\frac{1}{2}} = 6.44\ (Re\ Tu)^{0.066}$	2.59
(4) $Sh/Re^{\frac{1}{2}} = 8.04 + 0.171\ Re^{\frac{1}{2}}Tu + 0.191 \cdot 10^{-2}(Re^{\frac{1}{2}}Tu)^2$	1.92
(5) $Sh/Re^{\frac{1}{2}} = 8.04 + 0.568 \cdot 10^{-2}ReTu - 0.152 \cdot 10^{-5}(ReTu)^2$	1.59

TABLE 2. INFLUENCE OF TURBULENCE.

Authors	Relations	*ReTu* Range
This Work	$Sh = 0.549\ Re^{0.5}(ReTu)^{0.066}Sc^{1/3}$	$12<ReTu<600$
Lavender and Fei (1967)	$Nu = 2 + 0.717\ Re^{0.5}(ReTu)^{0.035}Pr^{1/3}$	$ReTu<1000$
	$Nu = 0 + 0.165\ Re^{0.5}(ReTu)^{0.250}Pr^{1/3}$	$ReTu>1000$
Yuge (1960) Correlated by Lavender and Pei (1967)	$Nu = 2 + 0.387\ Re^{0.5}(ReTu)^{0.085}Pr^{1/3}$	$ReTu<7000$
Gostkowski (1970)	$Nu = 1.431\ Re^{0.5}(ReTu)^{0.0214}Pr^{1/3}$	$ReTu<7000$
at the Stagnation Point	$Nu = 1.287\ Re^{0.5}(ReTu)^{0.2838}Pr^{1/3}$	$ReTu>7000$

good agreement with those recently presented by Sandoval et al., (1980), and obtained at $Tu = 0$ in another type of equipment where the sphere is moved in a quiescent liquid.

Correlations

The 240 data obtained in that work have been correlated using first a power law (Eq. 1), then the dimensionless groups $Re^{\frac{1}{2}}\ Tu$ (Eq. 2) proposed by Smith and Kuethe (1966) or $Re\ Tu$ (Eq. 3) used by Lavender and Pei (1967). More complicated equations have been checked (Eqs. 4 and 5) following Kestin and Wood (1971). The results are presented in Table 1.

It can be noted that for all these equations, the mean deviations between experimental and calculated values remain weak. Nevertheless this mean deviation decreases significantly for Eqs. 4 and 5 when the slope variations, observed in Figure 4, are taken into account.

Comparison with Previous Works

A detailed comparison with previous works is not easy because of differences in the forms of equations, or even because of the lack of correlations.

A first comparison with heat transfer data is presented in Table 2. It can be noted that in the experimental range covered by this work, $12 < Re\ Tu < 600$, the exponent of $Re\ Tu$ is low in all the equations proposed. The mass transfer results of this work lie between the heat transfer data of Lavender and Pei (1967) and those of Yuge (1960) reworked by Lavender and Pei (1967).

A larger comparison is shown on Figure 5 where the dimensionless groups $Sh/Re^{\frac{1}{2}}Sc^{1/3}$ or $Nu/Re^{\frac{1}{2}}Pr^{1/3}$ have been plotted as a function of the turbulent Reynolds number $Re\ Tu$. Again, it can be noted that, with the exception of Galloway and Sage (1967) all the authors have observed only a slight dependance of the transfer rate on the turbulent Reynolds number. A general agreement can also be observed for the values of $Sh/Re^{\frac{1}{2}}Sc^{1/3}$ or $Nu/Re^{\frac{1}{2}}Pr^{1/3}$ which lie in the range 0.6-0.8; note that the results of Gostkowski and Costello (1970), which are approximately twice these values, have been obtained not for the whole sphere but only at the stagnation point.

Finally the closest agreement with previous works has been found with the experimental results of Venezian et al. (1962) and the semi-empirical relation of Brauer (1973). For these two works, relative to heat and mass transfer between a gas and a sphere, the deviation with the present liquid results is less than 10% for the whole region of $Re\ Tu < 600$. This agreement suggests strongly that the influence of turbulence depends neither on the nature of the fluid phase nor on the transported quantity, Table 2.

NOTATION

D	=	sphere diameter
$\bar{v}$	=	time-smoothed velocity
v'	=	velocity fluctuation
X	=	distance from the porous plate
Λ	=	integral length scale of turbulence
Sc	=	Schmidt number
Sh	=	Sherwood number
Nu	=	Nusselt number
Pr	=	Prandtl number
Re	=	sphere Reynolds number
Tu	=	turbulence intensity

LITERATURE CITED

Baines, W. D., and E. G. Peterson, "An investigation of flow through screens," *Trans. Am. Soc. Mech. Eng.*, **73**, 467 (1951).

Bertrand, J., and J. P. Couderc, *Hot-film Probe Calibration in Liquids*, DISA info., **23**, 28 (1978).

Brauer, H., "Impuls-Stoff-und Wärmetransport durch die Grenzfloche Kugelförmiger Partikeln," *Chem. Ing. Tech.*, **45**, 1099 (1973).

Brown, R. A. S., K. Sato and B. H. Sage, *Material Transfer in Turbulent Gas Stream Effect of Turbulence on Macroscopie Transport from Spheres*, Chem. Eng. Data Series, 3, 263 (1958).

Brown, R. A. S., and B. H. Sage, "Material Transfer in Turbulent Gas Streams, Effect of Turbulence on Local Transport from Spheres," *J. Chem. Eng. Data*, **6**, 355 (1961).

Endoh, K., H. Tsuruga, H. Hirano, and M. Morihira, "Effect of Turbulence on Heat and Mass Transfer," *Heat Transfer*, Sap. Res., **1**, 113 (1972).

Frenkiel, F. N., "The Decay of Isotropic Turbulence," *Trans. Am. Soc. Mec. Eng.*, **70**, 311 (1948).

Galloway, T. R., and B. H. Sage, "Thermal and Material Transport from Spheres. Prediction of Macroscopic Thermal and Material Transport," *Int. J. Heat Mass Transfer*, **10**, 1195 (1967).

Galloway, T. R., and B. H. Sage, "Thermal and Material Transport from Spheres. Prediction of Local Transport," *ibid.*, **11**, 539 (1968).

Galloway, T. R., and D. M. Seid, "Inquiries into Turbulence Enhanced

Local Mass Transfer and Skin Friction from a Sphere using Electrochemical Probes," Letters in Heat Mass Transfer, 2, 247 (1975).
Gostkowski, V. J., and F. A. Costello, "The Effect of Free Stream Turbulence on Heat Transfer from the Stagnation Point of a Sphere," *Int. J. Heat Mass Transfer*, 13, 1382 (1970).
Hinze, J. D., *Turbulence* 2nd ed. McGraw Hill Book Co., New York (1975).
Hsu, N. T., and B. H. Sage, "Thermal and Material Transfer in Turbulent Gas Streams: Local Transport from Spheres," *AIChE J.*, 3, 405 (1957).
Kestin, J., and R. T. Wood, "The Influence of Turbulence on Mass Transfer from Cylinders," *J. Heat Transfer*, 93, 321 (1971).
Kolmogoroff, A. N., *On Degeneration of Isotropic Turbulence in an Incompressible Viscous Liquid*, C. R. Akad. Nauk. S.S.S.R., 31, 538 (1941).
Lavender, W.I., and D. C. T. Pei, "The Effect of Fluid Turbulence on the Rate of Heat Transfer from Spheres," *Int. J. Heat Mass Transfer*, 10, 529 (1967).
Maisel, D. S., and T. K. Sherwood, "Effect of Air Turbulence on Rate of Evaporation of Water," *Chem. Eng. Prog.*, 46, 131 (1950).
Mizushina, T., "Electrochemical Method in Transport Phenomena," *Advances in Heat Transfer*, p. 87, 7, Academic Press, New York, London (1970).
Raithby, G. D., and E. R. G. Eckert, "The Effect of Turbulence Parameters and Support Position on the Heat Transfer from Spheres," *Int. J. Heat Mass Transfer*, 11, 1233 (1968).
Sandoval, J. G., "Transfert de Matière entre une Sphère Solide et un Liquide. Influence de la Turbulence," *Thèse Doc. Ing.*, Inst. Nat. Polytech., Toulouse (1978).
Sandoval, J. G., J. P. Riba and J. P. Couderc, "Mass Transfer around a Sphere," *Trans. Inst. Chem. Eng.*, 58, 132 (1980).
Smith, M. C., and A. M. Kuethe, "Effects of Turbulence on Laminar Skin Friction and Heat Transfer," *Phys. Fluids*, 9, 2337 (1966).
Venezian, E., M. J. Crespo and B. H. Sage, "Thermal and Material Transfer in Turbulent Gas Streams: One inch Spheres," *AIChE J.*, 8, 383 (1962).
Yuge, T., "Experiments on Heat Transfer of Spheres, Including Combined Natural and Forced Convection," *J. Heat Transfer*, 82, 214 (1960).

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Revision of Kynch Sedimentation Theory

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Much of the theory of gravity sedimentation has been based on the work of Coe and Clevenger (1916) and Kynch (1952). They provided methods for obtaining rates of sedimentation in batch, static tests which are presently used for design of continuous thickeners. Kynch assumed that a first order partial differential equation controlled the entire sedimentation process. His equation was based on: (1) continuity balance; and (2) sedimentation velocity being a unique function of solid particulate concentration. A general solution was presented in the form of volume fraction of solids $\phi_s = f(x - \nu t)$. During the constant rate fall of the upper interface, the boundary condition of uniform initial concentration combines with the Kynch equations to adequately describe the sedimentation phenomena. Kynch ignored the sediment rising from the bottom of the settling chamber, and assumed that the characteristics $y = x - \nu t$ originated at the origin of coordinates (height, time) during the first falling rate period. The characteristics actually originate at the surface of the rising sediment where the upward liquid velocity affects the rate of fall of the particulates. New equations have been derived based upon the assumption that the characteristics emanate from the rising sediment.

SCOPE

Gravity sedimentation is used to thicken slurries prior to filtration or centrifugation. Normally a larger fraction of the total liquid is removed in thickening than in subsequent operations. Continuous thickeners involve a clear zone of small turbidity, a region of line settling of particles, and a sediment zone. In the clear zone, individual particles rise or fall in a Stokesian regime. Under ideal conditions, particles in line or hindered settling have uniform velocities independent of particle size distribution. The sediment or compression zone involves a structural network of particles capable of sustaining compressive forces. The particulate structure has a low yield stress and compresses readily under the weight of the buoyed particles. The compressive or effective pressure due to the sediment weight increases with depth leading to a decrease in porosity.

Motion of rakes, introduction of feed in a center feedwell, and methods of removal of the overflow and underflow contribute to three-dimensional flow patterns which complicate actual behavior of industrial thickeners.

For many years, design of gravity thickeners has been based upon the work of Coe and Clevenger (1916), and Kynch (1952) as modified by Talmadge and Fitch (1955) and discussed at length by Scott (1966, 1968) and Fitch (1966, 1977). The Kynch procedure is appealing because it permits the velocity of sedimentation u_s and the volumetric flux/area, $F = u_s\phi_s$ to be determined as functions of a range of volumetric fraction of solids ϕ_s in a single run. The Coe and Clevenger method requires preparation of slurries of different concentration whose rates of sedimentation are determined separately thus requiring more time and possibly larger samples than with the Kynch technique. In addition when flocculants are used, the Kynch procedure is attractive because time is diminished and